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Symmetric reggeon interaction in perturbative QCD

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Abstract

Integral kernels describing the pair interaction of reggeized gluons and quarks are reconstructed in terms of conformal symmetric 4-point functions in the transverse plane of impact parameters.

1 Introduction

The conformal symmetry emerging in the leading order BFKL equation [1] played a major role in its solution [2] and in a number of related investigations. The symmetry is useful in formulating the multiple reggeized gluon exchange exhibiting integrability properties [3, 4]. Remarkable symmetry features of the gluonic reggeon interaction have been pointed out [5] and relevant methods of integrable systems have been adapted to the problem [6, 7, 8, 9]. This symmetry has been observed also in the interaction of reggeized quarks with each other and with the leading reggeized gluons [10].

In the BFKL pomeron transition vertices [11, 12] the conformal symmetry structure is an important aspect [13, 14, 15, 18]. The role of this symmetry in the dipole picture [16, 17] and its relation to the BFKL formulation has been shown e.g. in the papers [19, 20, 21].

The leading order reggeized gluon interaction and the triple vertex of BFKL pomerons have been analyzed recently with respect to their conformal symmetry properties [22], discussing also the relation to the dipole picture and the Balitsky-Kovchegov equation.

In the generalized leading logarithmic approximation the exchanged reggeised QCD quanta, called reggeons in the following, appear as particles moving in transverse (impact parameter) space interacting pairwise. The symmetry acts as the Möbius transformation on the (complex valued) position variable running over the transverse plane,

$$z \to z' = \frac{az+b}{cz+d}, \quad z \to z^{*\prime} = \frac{\tilde{a}z+\tilde{b}}{\tilde{c}z+\tilde{d}} \quad . \tag{1.1}$$

In view of the symmetry of the interaction the reggeon states can be described by holomorphic and anti-holomorphic weights $(\ell) = (\ell, \tilde{\ell})$ characterizing the conformal representations, or by the scaling dimension $\ell + \tilde{\ell}$ and the spin $\ell - \tilde{\ell} = [\ell]$. The states of a single leading gluonic reggeon are characterized by the weights $(\ell) = (0,0)$ and the fermionic reggeons by $(\ell) = (\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$ in dependence on their s-channel helicity. The interaction of fermionic reggeons of opposite helicity results in double-logarithmic contributions. These contributions can be accounted for by displacing in this anti-parallel helicity configuration the weights as $(\ell) = (\frac{\omega}{4}, \frac{1}{2} - \frac{\omega}{4})$ or $(\frac{1}{2} - \frac{\omega}{4}, \frac{\omega}{4})$ [10]. Here ω is the complex angular momentum i.e. the variable conjugated to the energy squared s in the Mellin transformation defining the partial waves in the Regge asymptotics.

The states corresponding to the principal series of representations of $SL(2, C)$ have integer or half-integer values of spin n and scaling dimension $1 + 2i\nu$ with ν real, $(\ell) = (\frac{1+n}{2} + i\nu, \frac{1-n}{2} + i\nu)$.

In the present paper we reconstruct the pairwise reggeon interaction relying on this symmetry. We represent the interaction in terms of integral operators with integrations over the transverse impact parameter plane. The input are the mentioned conformal weights of the one-reggeon states and the eigenvalues of the QCD one-loop interaction operators on two-reggeon states corresponding to the pricipal series. The latter are well known for the BFKL (gluon-gluon) case [1] and have been obtained in [10] for the cases involving fermionic reggeons by solving the corresponding version of BFKL-type equations in momentum representation in the forward kinematics.

This study was motivated in particular by previous studies concerning the Bjorken asymptotics, where the one-loop parton interaction or leading-twist composite operator renormalization has been represented in terms of conformal symmetric operators [23, 24].

2 Symmetric correlators and operators

2.1 Correlators

We are going to build the symmetric interaction operators with integral kernels being symmetric 4-point functions. Symmetric n-point functions with the weights (ℓ_i) corresponding to the point x_i obey

$$\sum_i^n S^a_{i,\ell_i}\ G(x_1, ..., x_n) = 0, \quad \sum_i^n \tilde{S}^a_{i,\ell_i}\ G(x_1, ..., x_n) = 0. \tag{2.1}$$

The generators of the holomorphic Möbius transformation (1.1) are

$$S^-_{i,0} = \partial_i, \quad S^0_{i,0} = x_i \partial_i, \quad S^+_{i,0} = -x_i^2 \partial_i, \tag{2.2}$$

and the ones of the weight ℓ representation are

$$S^-_{i,\ell} = S^-_{i,0}, \quad S^0_{i,\ell} = x_i^{-\ell} S^0_{i,0} x_i^\ell, \quad S^+_{i,0} = x_i^{-2\ell} S^+_{i,0} x_i^{2\ell}. \tag{2.3}$$

The anti-holomorphic generators $\tilde{S}^a_{i,\ell}$ have the analogous form with the weight ℓ replaced by $\tilde{\ell}$ and the holomorphic variables x_i and derivatives ∂_i replaced by the anti-holomorphic ones. The symmetry conditions (2.1) are solved by a power-like expression in the differences of coordinates $x_{ij} = x_i - x_j$

$$\prod_{i<j} x_{ij}^{a_{ij}} \quad x_{ij}^{*\tilde{a}_{ij}} \tag{2.4}$$

with the exponents ($a_{ij} = a_{ji}$) obeying

$$\sum_{j=1}^{n} a_{ij} = -2\ell_i, \quad \sum_{j=1}^{n} \tilde{a}_{ij} = -2\tilde{\ell}_i. \tag{2.5}$$

For $n = 2, 3, 4$ this includes the well-known facts that the 3-point functions are in general uniquely defined by the weights, that 2-point functions are non-vanishing for coinciding weights only, and that the 4-point functions are determined by the weights up to an arbitrary function of the anharmonic ratio. The latter case $n = 4$ is of particular interest in our context. We denote the index values i, j by $1, 2, 1', 2'$ and the corresponding weights by $\ell_1, \ell_2, \bar{\ell}_{1'}\bar{\ell}_{2'}$. We parametrize the exponents as

$$\begin{aligned} a_{12} = d + \frac{\sigma}{2} - \ell_1 - \ell_2, \quad a_{1'2'} = d + \frac{\sigma}{2} - \bar{\ell}_{1'} - \bar{\ell}_{2'}, \\ a_{12'} = h + \frac{\sigma}{2} - \ell_1 - \bar{\ell}_{2'}, \quad a_{1'2} = h + \frac{\sigma}{2} - \bar{\ell}_{1'} - \ell_2, \\ a_{11'} = -d - h - \ell_1 - \bar{\ell}_{1'}, \quad a_{22'} = -d - h - \ell_2 - \bar{\ell}_{2'}, \\ \sigma = \ell_1 + \ell_2 + \bar{\ell}_{1'} + \bar{\ell}_{2'}. \end{aligned} \tag{2.6}$$

The analogous relations hold for the exponents $\tilde{a}_{ij}$ of the anti-holomorphic powers. Here and in the following expressions typically consist of holomorphic and anti-holomorphic parts and we follow [7] in using the short-hand notations

$$\begin{aligned} (\alpha) = (\alpha, \tilde{\alpha}), \quad [x]^{(\alpha)} = x^{\alpha} \cdot x^{*\tilde{\alpha}}, \\ [\alpha] = \alpha - \tilde{\alpha}, \quad a(\alpha) = \frac{\Gamma(1-\tilde{\alpha})}{\Gamma(\alpha)}. \end{aligned} \tag{2.7}$$

If the entries in the doublet (α) are equal numbers we shall sometimes write e.g. (1) instead of (1,1).

The expressions (2.4), (2.5, (2.6) define single-valued functions of the complex variables x_i if $[a_{ij}]$ are integers.

The dependence of the 4-point functions on the two doublets of parameters $(d), (h)$ enters as exponents of anharmonic ratios,

$$r_{ts}^{-d-h} \cdot r_{tu}^{h}, \quad r_{ts} = \frac{x_{11'}x_{22'}}{x_{12}x_{1'2'}}, \quad r_{tu} = \frac{x_{12'}x_{21'}}{x_{12}x_{1'2'}}. \tag{2.8}$$

Because these ratios are dependent,

$$r_{tu} = r_{ts} - 1, \tag{2.9}$$

a generic form of the symmetric 4-point fucntion can be represented by linear combinations of (2.4), (2.6) with only one doublet of the parameters $(d),(h)$ varying.

In the particular cases of some exponents a_{ij} being negative integers besides of the monomials (2.4) we have other expressions obeying the same symmetry conditions (2.1). For example, if $(a_{11'}) = (-1)$ then the expression with the factor $|x_{11'}^2|^{-1}$ replaced by $\delta^{(2)}(x_{11'})$ has the same behaviour under conformal transformations.

2.2 Operators

We are going to construct symmetric operators using the symmetric n-point functions as integral kernels.

The point about negative integer exponents mentioned above shows up also in the construction of operators. This is illustrated in the simple example of the 2-point function with $(\ell_1) = (\ell_{1'}) = (\frac{1-\varepsilon}{2})$ acting as the operator kernel on $f(x)$.

$$\int \frac{d^2 x_{1'}}{|x_{11'}|^{2-2\varepsilon}} f(x_{1'}) = \frac{\pi}{\varepsilon} f(x_1) + \int \frac{d^2 x_{1'}}{|x_{11'}|^2}(f(x_{1'}) - f(x_1)) + \mathcal{O}(\varepsilon) \qquad (2.10)$$

We find the decomposition rule in the limit,

$$\frac{1}{|x_{11'}|^{2-2\varepsilon}} = \frac{\pi}{\varepsilon}\delta^{(2)}(x_{11'}) + \frac{1}{|x_{11'}^2|_+} + \mathcal{O}(\varepsilon). \qquad (2.11)$$

In the limit $\varepsilon \to 0$ we have two operators (acting on one-reggeon wave functions) with the same symmetry properties, one of them is just the identity operator.

Consider also the operator defined by the 2-point function with $(\ell_1) = (\ell_{1'}) = (-1 + a + \frac{n}{2}, -1 + a - \frac{n}{2})$, n integer. Acting on the function of the form $\psi_{b,m}(x) = x^m |x|^{-2+2b-m}$, m integer, one obtains a similar function with shifted parameters, $\psi_{a+b,m+n}$,

$$\begin{aligned}\int d^2 x_{1'} \frac{x_{11'}^n}{|x_{11'}|^{2-2a+n}} \frac{x_{1'}^m}{|x_{1'}|^{2-2b+m}} &= \pi \frac{x_1^{n+m}}{|x_1|^{2-2(a+b)+n+m}} \\ &\frac{\Gamma(1-a-b+\frac{n+m}{2})\, \Gamma(a+\frac{n}{2})\, \Gamma(b+\frac{m}{2})}{\Gamma(1-a+\frac{n}{2})\, \Gamma(1-b+\frac{m}{2})\, \Gamma(a+b+\frac{n+m}{2})}.\end{aligned} \qquad (2.12)$$

The operator is defined by the integral for $a > 0, b > 0, a + b < 1$. For other values of the parameters we define the operator by analytic continuation provided by the right-hand side. In terms of the short-hand notation (2.7) the latter relation can be written as

$$\int d^2 x_{1'}\ [x_{11'}]^{(-\alpha)}\ [x_{1'}]^{-(\beta)} = \pi a(\alpha)\ a(\beta)\ a(2 - \tilde{\alpha} - \tilde{\beta})\ [x_1]^{(-\alpha-\beta+1)} \qquad (2.13)$$

with $(\alpha) = (1 - a + \frac{n}{2}, 1 - a - \frac{n}{2}), (\beta) = (1 - b + \frac{m}{2}, 1 - b - \frac{m}{2})$. A more general form of this relation is obtained by shifting the points $x_1, x_{1'}$ by x_2 and doing the inversion about an arbitrary point x_3.

$$\int d^2 x_{1'}\ [x_{11'}]^{(-\alpha)}\ [x_{21'}]^{(-\beta)}\ [x_{31'}]^{(-\gamma)}\ =$$

$$\pi a(\alpha)\ a(\beta)\ a(\gamma)\ [x_{12}]^{(\gamma-1)}\ [x_{23}]^{(\alpha-1)}\ [x_{31}]^{(\beta-1)}, \\ (\alpha+\beta+\gamma)=(2) \tag{2.14}$$

We turn to the operators acting on two-reggeon states, i.e. on functions of two complex variables x_1, x_2, the behaviour under conformal transformations of which is characterized by the weights $(\ell_1), (\ell_2)$. Two-reggeon states of definite weight (ℓ_0) are represented by the 3-point functions

$$E^{(\ell_0)}_{(\ell_1),(\ell_2)}(x_1,x_2;x_0) = [x_{12}]^{(\ell_0-\ell_1-\ell_2)}\ [x_{10}]^{(\ell_2-\ell_1-\ell_0)}\ [x_{20}]^{(\ell_1-\ell_2-\ell_0)}. \tag{2.15}$$

The position variable x_0 serves as a label of states within the representation (ℓ_0).

We consider operators acting symmetrically from the tensor product representation $V_{(\ell_{1'})}\otimes V_{(\ell_{2'})}$ to $V_{(\ell_1)}\otimes V_{(\ell_2)}$. Then their kernels are symmetric 4-point functions (2.4), (2.5) with the weights $(\ell_1), (\ell_2), (\bar\ell_{1'}) = (1-\ell_{1'}), (\bar\ell_{2'}) = (1-\ell_{2'})$.

$E^{(\ell_0)}_{(\ell_1),(\ell_2)}(x_1,x_2;x_0)$ are eigenfunctions of these operators in the case $(\ell_1) = (\ell_{1'}), (\ell_2) = (\ell_{2'})$. Generic symmetric operators with these weights can be represented in terms of these kernels by variation of only one doublet of parameter out of the two $(d), (h)$. If these parameters are related by $(d+h+1) = (0)$ at $(\ell_1) = (\ell_{1'}), (\ell_2) = (\ell_{2'})$ the integrand simplifies by vanishing of the exponents of $x_{11'}, x_{22'}$. In this particular case we denote the kernel by $K^{(d)}_{(\ell_1),(\ell_2)}(x_1,x_2;x_{1'},x_{2'})$ and the corresponding operator by $\hat K^{(d)}_{(\ell_1),(\ell_2)}$. The eigenvalue relation reads

$$\begin{aligned} \hat K^{(d)}_{(\ell_1),(\ell_2)} E^{(\ell_0)}_{(\ell_1),(\ell_2)}(x_1,x_2;x_0) &= \\ \frac{1}{\pi^2}\int d^2x_{1'}d^2x_{2'}K^{(d)}_{(\ell_1),(\ell_2)}(x_1,x_2;x_{1'},x_{2'})E^{(\ell_0)}_{(\ell_{1'}),(\ell_{2'})}(x_1,x_2;x_0) &= \\ \lambda^{(d)}_{(\ell_1),(\ell_2),(\ell_0)}\ E^{(\ell_0)}_{(\ell_1),(\ell_2)}(x_1,x_2;x_0) & \end{aligned} \tag{2.16}$$

The eigenvalues are calculated by applying (2.14) (compare [7])

$$\lambda^{(d)}_{(\ell_1),(\ell_2),(\ell_0)} = (-1)^{[d+\ell_1-\ell_2]}\ a(1+d-\ell_1+\ell_2)a(1+d+\ell_1-\ell_2)\ a(1-d-\ell_0)a(-d+\ell_0). \tag{2.17}$$

Notice that the product $\lambda^{(d)}_{(\ell_1),(\ell_2),(\ell_0)}\ \lambda^{(-d)}_{(\ell_1),(\ell_2),(\ell_0)}$ does not depend on (ℓ_0). Therefore the product of the corresponding operators is the identity up to normalization (on the space of functions spanned by (2.15) with (ℓ_0) taking the values of the principal series),

$$\hat K^{(d)}_{(\ell_1),(\ell_2)}\ \hat K^{(-d)}_{(\ell_1),(\ell_2)} = \rho(\ell_1,\ell_2,d)\ \hat I. \tag{2.18}$$

Moreover, $\hat K^{(d)}_{(\ell_1),(\ell_2)}$ obeys the Yang-Baxter relation with (d) playing the role of the spectral parameter [7].

We expand the (ℓ_0) dependent factors in the eigenvalues at $(d) = (\Delta) - \varepsilon$,

$$\begin{aligned} a(1-\Delta-\ell_0-\varepsilon)\ a(-\Delta+\ell_0-\varepsilon) = a(1-\Delta-\ell_0)\ a(-\Delta+\ell_0)\cdot \\ \left\{1+\varepsilon\ [\ \chi_{-\Delta}(\ell_0(1-\ell_0)) + \chi_{\tilde\Delta}(\tilde\ell_0(1-\tilde\ell_0)) - 4\psi(1)] + \mathcal{O}(\varepsilon^2)\right\} \end{aligned} \tag{2.19}$$

Here we have used the notation

$$\chi_\Delta(x) = \psi(1) - \psi(x+\Delta) - \psi(1-x+\Delta) \tag{2.20}$$

which appeared in [10] in writing the one-loop results for the eigenvalues of the QCD reggeon interactions with (Δ) taking the particular values $(\Delta) = \pm(\ell_1 - \ell_2)$.

We notice further that the factors in the eigenvalues (2.17) that do not depend on (ℓ_0) have a pole in ε at $(d) = \pm(\ell_1 - \ell_2) - \varepsilon$. This singularity is caused by some exponents in the kernel approaching negative integers, indeed

$$(a_{12'}) = (-1 - d - \ell_1 + \ell_2), \quad (a_{21'}) = (-1 - d + \ell_1 - \ell_2). \tag{2.21}$$

From the above discussion it is clear that in this case the definition of regular symmetric operators takes some care, following the example of a one-point operator (2.10) (2.11). This will be done for the physically relevant cases in the following.

3 Identical reggeons

Consider now the case $(\ell_1 - \ell_2) = (0)$ relevant for the interaction of two gluonic reggeons (BFKL), $(\ell_1) = (\ell_2) = (0)$, or of two fermionic reggeons of parallel helicity, $(\ell_1) = (\ell_2) = (\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$. At $(d) = -\varepsilon$ we have

$$\lambda^{(-\varepsilon)}_{(\ell_1),(\ell_2),(\ell_0)} = \frac{(-1)^{[\ell_0]}}{\varepsilon^2} \Big\{ 1 + \varepsilon \; [\; \chi_0(\ell_0(1-\ell_0)) + \chi_0(\tilde{\ell}_0(1-\tilde{\ell}_0))] + \mathcal{O}(\varepsilon^2) \Big\} \tag{3.1}$$

The coefficient of ε^{-1} is proportional to the well-known BFKL pomeron eigenvalues [1, 2] with $\ell_0 = \frac{1-n}{2} + i\nu, [\ell_0] = n$. The exponents of the kernels (2.6) at $(d) = (-\varepsilon)$ are

$$(a_{12'}) = (a_{21'}) = (-1+\varepsilon), \; (a_{1'2'}) = (-1-\varepsilon+\ell_1+\ell_2), \; (a_{12}) = (1-\varepsilon-\ell_1-\ell_2). \tag{3.2}$$

Consider first the case $(\ell_1) = (\ell_2) = (\ell) \neq 0$, where the singularity of the type (2.10) (2.11) appears twice. In the limit $\varepsilon \to 0$ the 4-point function (2.4) and the ones with some of the negative integer powers replaced by δ-functions behave equally under conformal transformations. We find combinations of these 4-point functions that define regular operators.

Replacing both $|x^2_{12'}|^{-1}$ and $|x^2_{21'}|^{-1}$ by the corresponding δ-functions we obtain the permutation operator $\hat{P}_{12}$ with the kernel $\delta^{(2)}(x_{12'}) \; \delta^{(2)}(x_{21'})$. Replacing instead only one of these negative-integer powers by the corresponding δ-function we define the symmetric operator $\hat{K}^{0+}_{(\ell_1)}$ with the kernel

$$K^{0+}_{(\ell)} = [x_{12}]^{(1-2\ell)} \; [x_{1'2'}]^{(-1+2\ell)} \\ \left\{ \frac{1}{|x^2_{21'}|_+} \delta^{(2)}(x_{12'}) + \frac{1}{|x^2_{12'}|_+} \delta^{(2)}(x_{21'}) \right\} \tag{3.3}$$

Approaching $(d) = (0)$ with the generic operator $\hat{K}^{(-\varepsilon)}_{(\ell),(\ell)}$ we have the expansion in terms of the latter two operators,

$$\hat{K}^{(-\varepsilon)}_{(\ell),(\ell)} = A(\varepsilon)\hat{P}_{12} + B(\varepsilon)K^{0+}_{(\ell)},$$
$$A(\varepsilon) = \varepsilon^{-2}(\pi^2 + \mathcal{O}(\varepsilon)), \quad B(\varepsilon) = \varepsilon^{-1}(\pi + \mathcal{O}(\varepsilon)). \tag{3.4}$$

We identify the operator $\hat{P}_{12} \cdot \hat{K}^{0+}_{(\ell)}$ as the one representing the identical reggeon interaction for $(\ell_1) = (\ell_2) = (\ell) \neq 0$, in particular the parallel helicity fermionic reggeons, Fig 1a. Its kernel is obtained from (3.3) by interchanging the labels $1', 2'$. Its eigenvalues on the two-reggeon functions (2.15) are

$$\chi_0(\ell_0(1-\ell_0)) + \chi_0(\tilde{\ell}_0(1-\tilde{\ell}_0)) \tag{3.5}$$

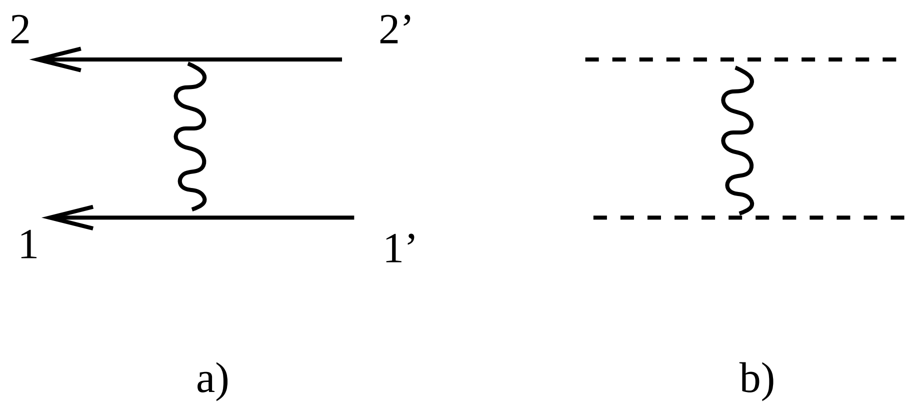

Figure 1: Identical reggeon interaction. a) fermions of parallel helicity, b) gluonic reggeons (dashed lines).

In the case $(\ell_1) = (\ell_2) = (0)$ simultaneously with $(a_{12'}) = (a_{21'})$ also $(a_{1'2'})$ approaches (-1). On two-reggeon wave functions vanishing at coinciding points $x_1 = x_2$ a regular operator can be defined by a modified combination of the ones above obtained by the replacement of the powers $|x^2_{12'}|^{-1}$ and $|x^2_{21'}|^{-1}$ by δ-functions. The corresponding kernel can be written in terms of an integral over the auxiliary point $x_{3'}$ as

$$K^{0+}_{(0)} = \int \frac{d^2x_{3'}|x^2_{12}|}{|x^2_{13'}||x^2_{23'}|} \cdot$$
$$\Big(\delta^{(2)}(x_{21'})\,\delta^{(2)}(x_{2'3'}) \;+\; \delta^{(2)}(x_{13'})\,\delta^{(2)}(x_{1'3'}) \;-\; \delta^{(2)}(x_{12'})\,\delta^{(2)}(x_{21'})\Big). \tag{3.6}$$

With the operator defined by this kernel the expansion (3.4) holds analogously. The operator $\hat{P}_{12} \cdot \hat{K}^{0+}_{(0)}$ represents the gluonic reggeon interaction, Fig. 1b. Its kernel is obtained from (3.6) by interchanging the labels $1', 2'$ and its eigenvalues are given by (3.5).

The resulting kernel is the one known in the dipole picture of BFKL [17]. This representation of the BFKL kernel and its relation to other ones has been studied recently [22].

4 Fermion - gluon interaction

The pair interaction of fermionic and gluonic reggeons in an overall colour singlet exchange is determined by a symmetric operator with the weights $(\ell_1) = (\frac{1}{2}, 0), (\ell_2) = (0,0)$. The corresponding kernel $K^{(d)}_{(\ell_1),(\ell_2)}$ is single-valued for $[d] = \frac{1}{2} + m$, m integer, and the wave functions (2.15) are single-valued for $[\ell_0] = \frac{1}{2} + n$, n integer. At $(d) = (\ell_1 - \ell_2 - \varepsilon)$ the eigenvalues (2.17) behave like

$$\lambda^{(\ell_1-\ell_2-\varepsilon)}_{(\ell_1),(\ell_2),(\ell_0)} = -\frac{(-1)^n}{\varepsilon^2}(\ell_0 - \frac{1}{2}) \\ \Big\{1 + \varepsilon\ [\ \chi_{-\frac{1}{2}}(\ell_0(1-\ell_0)) + \chi_0(\tilde{\ell}_0(1-\tilde{\ell}_0)) + 1] + \mathcal{O}(\varepsilon^2)\Big\} \tag{4.1}$$

and at $(d) = (-\ell_1 + \ell_2 - \varepsilon)$ we obtain

$$\lambda^{(-\ell_1+\ell_2-\varepsilon)}_{(\ell_1),(\ell_2),(\ell_0)} = \frac{(-1)^n}{\varepsilon}(\ell_0 - \frac{1}{2})^{-1} \\ \Big\{1 + \varepsilon\ [\ \chi_{\frac{1}{2}}(\ell_0(1-\ell_0)) + \chi_0(\tilde{\ell}_0(1-\tilde{\ell}_0))] + \mathcal{O}(\varepsilon^2)\Big\}. \tag{4.2}$$

At $(d) = (\ell_1 - \ell_2 - \varepsilon)$ we encounter two exponents (2.6) approaching negative integers, $(a_{12'}) = (-2,-1) + (\varepsilon)$, $(a_{12'}) = (-1,-1) + (\varepsilon)$. We have the regular symmetric operators defined by the kernels

$$K^{+(\frac{1}{2},0),\delta} = -x_{12}\ \ \partial_{2'}\ \delta^{(2)}(x_{12'}) \cdot \delta^{(2)}(x_{21'}), \\ K^{+(\frac{1}{2},0),+} = |x_{12}^2| \frac{x_{1'2'}}{|x_{1'2'}^2|}\left\{\partial_{2'}\delta^{(2)}(x_{12'})\frac{1}{|x_{21'}^2|_+}\ +\ \partial_{2'}\frac{1}{|x_{12'}^2|_+}\delta^{(2)}(x_{21'})\right\}, \tag{4.3}$$

and the decomposition

$$\hat{K}^{(\frac{1}{2}-\varepsilon,-\varepsilon)}_{(\frac{1}{2},0),(0,0)} = A_+(\varepsilon)\hat{K}^{(\frac{1}{2},0),\delta}\ +\ B_+(\varepsilon)\ \hat{K}^{(\frac{1}{2},0),+}, \\ A_+(\varepsilon) = \frac{1}{\varepsilon^2}(\pi^2 + \mathcal{O}(\varepsilon),\quad B_+(\varepsilon) = \frac{1}{\varepsilon}(\pi + \mathcal{O}(\varepsilon). \tag{4.4}$$

At $(d) = (-\ell_1 + \ell_2 - \varepsilon)$ we have the following exponents approaching negative integers, $(a_{12'} = (a_{1'2'}) = (-1+\varepsilon)$. On wave functions vanishing at coinciding arguments, $x_1 = x_2$, we have regular symmetric operators given by the kernels

$$K^{-(\frac{1}{2},0),\delta} = x_{12}^*\ |x_{1'2'}^2|^{-1}\ \delta^{(2)}(x_{12'})\ \frac{x_{21'}}{|x_{21'}^2|}, \\ K^{-(\frac{1}{2},0),+} = x_{12}^*\ |x_{1'2'}^2|^{-1}\ \frac{1}{|x_{12'}^2|_+}\ \frac{x_{21'}}{|x_{21'}^2|}, \tag{4.5}$$

and the decomposition

$$\hat{K}^{(-\frac{1}{2}-\varepsilon,-\varepsilon)}_{(\frac{1}{2},0),(0,0)} = A_-(\varepsilon)\hat{K}^{-(\frac{1}{2},0),\delta}\ +\ B_-(\varepsilon)\ \hat{K}^{-(\frac{1}{2},0),+}, \\ A_-(\varepsilon) = \frac{1}{\varepsilon}(\pi + \mathcal{O}(\varepsilon),\quad B_-(\varepsilon) = (1 + \mathcal{O}(\varepsilon). \tag{4.6}$$

The operator composed out of the above ones as

$$-\left(\hat{K}^{(-\frac{1}{2},0),\delta}\ \hat{K}^{(\frac{1}{2},0),+}\ +\ \hat{K}^{(+\frac{1}{2},0),\delta}\ \hat{K}^{(\frac{1}{2},0),-}\right) \tag{4.7}$$

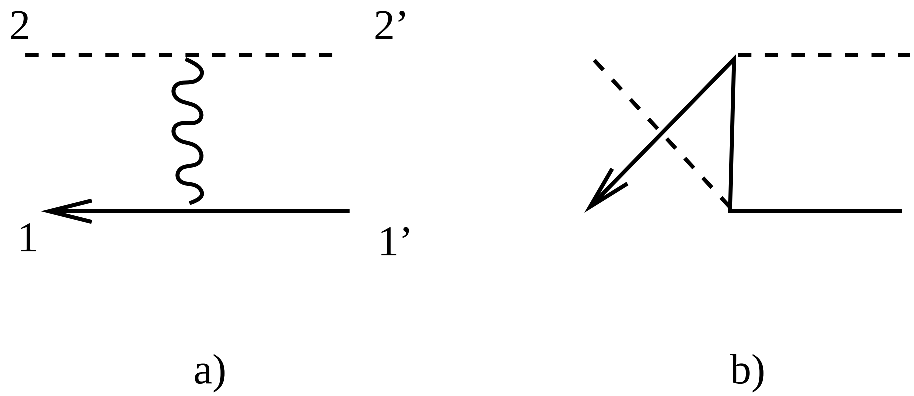

Figure 2: Interaction of reggeized fermion and gluon, a) by gluon exchange, b) by fermion exchange.

has the eigenvalues on the two-reggeon wave functions (2.15)

$$2\chi_0(\tilde\ell_0(1-\tilde\ell_0)) \ + \ \chi_{\frac{1}{2}}(\ell_0(1-\ell_0)) \ + \ \chi_{-\frac{1}{2}}(\ell_0(1-\ell_0)) \ + \ 1 \tag{4.8}$$

and in this way represents (up to normalization) the fermion-gluon reggeon interaction mediated by an s-channel gluon, Fig. 2a.

We notice also that the operator $\hat{K}^{(-\frac{1}{2},0),\delta}$ having the eigenvalues on the two-reggeon wave functions (2.15) for $\tilde\ell_0 = \ell_0 + \frac{1}{2} + n$ equal to $(-1)^n(\ell_0 - \frac{1}{2})^{-1}$ represents the fermion-gluon interaction mediated by a fermion, Fig. 2b.

5 Fermions of anti-parallel helicities

Consider first the formal case $(\ell_1) = (\frac{1}{2}, 0), (\ell_2) = (0, \frac{1}{2})$, ignoring double-log contributions. At $(d) = (\pm\ell_1 \mp \ell_2 - \varepsilon)$ the eigenvalues (2.17) are

$$\lambda^{(\pm\ell_1\mp\ell_2-\varepsilon)}_{(\ell_1),(\ell_2),(\ell_0)} = \frac{1}{\varepsilon}\frac{\Gamma(\tilde\ell_0 \mp \frac{1}{2})\Gamma(1-\tilde\ell_0 \mp \frac{1}{2})}{\Gamma(1-\ell_0 \mp \frac{1}{2})\Gamma(\ell_0 \mp \frac{1}{2})}$$
$$\left\{1 + \varepsilon\ [\ \chi_{\mp\frac{1}{2}}(\ell_0(1-\ell_0)) + \chi_{\mp\frac{1}{2}}(\tilde\ell_0(1-\tilde\ell_0)) + 1] + \mathcal{O}(\varepsilon^2)\right\} \tag{5.1}$$

At $(d) = (\pm\ell_1 \mp \ell_2) = (\pm\Delta)$ the exponent $(a_{21'})$ or $(a_{12'})$ turns to (-1) and we have the regular symmetric operator kernels obtained from $K^{(d)}_{(\ell_1),(\ell_2)}$ by replacing the factor with the exponent -1, $|x_{ij}^2|^{-1}$, by $\delta^{(2)}(x_{ij})$ or by $|x_{ij}^{-2}|_+$. In this way we are led to define the kernels

$$K^{\frac{1}{2},\delta} = x_{12}\ \frac{x_{1'2'}}{|x_{1'2'}|^2}\ \ \delta^{(2)}(x_{21'})\ \frac{x^{*2}_{12'}}{|x^4_{12'}|},$$
$$K^{\frac{1}{2},+} = x_{12}\ \frac{x_{1'2'}}{|x_{1'2'}|^2}\ \ \frac{1}{|x^2_{21'}|_+}\ \frac{x^{*2}_{12'}}{|x^4_{12'}|},$$
$$K^{-\frac{1}{2},\delta} = x^*_{12}\ \frac{x^*_{1'2'}}{|x_{1'2'}|^2}\ \ \delta^{(2)}(x_{12'})\frac{x^2_{21'}}{|x^4_{21'}|},$$

$$K^{-\frac{1}{2},+} = x^*_{12}\ \frac{x_{1'2'}}{|x_{1'2'}|^2}\ \ \frac{1}{|x^2_{12'}|_+}\ \frac{x^2_{21'}}{|x^4_{21'}|}. \tag{5.2}$$

The operator $\hat{K}^{(d)}_{(\ell_1),(\ell_2)}$, with $(\ell_1) = (\frac{1}{2}, 0), (\ell_2) = (0, \frac{1}{2})$ decomposes at $(d) = \pm(\frac{1}{2}, -\frac{1}{2}) - (\varepsilon)$ as

$$\hat{K}^{(\pm\frac{1}{2}-\varepsilon,\mp\frac{1}{2}-\varepsilon)}_{(\frac{1}{2},0),(0,\frac{1}{2})} = A(\varepsilon)\hat{K}^{\pm\frac{1}{2},\delta}\ +\ B(\varepsilon)\ \hat{K}^{\pm\frac{1}{2}+}\ ,$$
$$A(\varepsilon) = \frac{1}{\varepsilon}(\pi + \mathcal{O}(\varepsilon),\quad B(\varepsilon) = (1 + \mathcal{O}(\varepsilon). \tag{5.3}$$

The operator composed out of the ones defined by the above kernels (5.2) as

$$\hat{K}^{-\frac{1}{2},\delta}\ \hat{K}^{+\frac{1}{2}+}\ +\ \hat{K}^{+\frac{1}{2},\delta}\ \hat{K}^{-\frac{1}{2},+} \tag{5.4}$$

has the eigenvalues on the two-reggeon wave functions (2.15)

$$\chi_{-\frac{1}{2}}(\tilde{\ell}_0(1-\tilde{\ell}_0)) + \chi_{\frac{1}{2}}(\tilde{\ell}_0(1-\tilde{\ell}_0)) + \chi_{\frac{1}{2}}(\ell_0(1-\ell_0)) + \chi_{-\frac{1}{2}}(\ell_0(1-\ell_0)) + 2. \tag{5.5}$$

It describes in leading $\ln s$ accuracy the interaction of anti-parallel helicity fermionic reggeons, Fog. 3, besides of the two-reggeon states with $[\ell_0] = 0$, where double-log contributions appear.

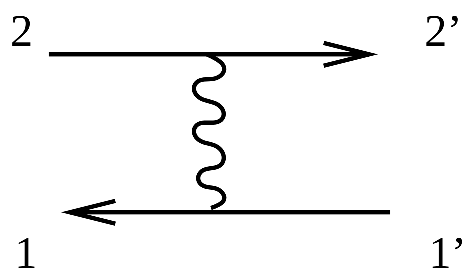

Figure 3: Interaction of reggeized fermions of anti-parallel helicities.

In the case $(\ell_1) = (\frac{1}{2} - \frac{\omega}{4}, \frac{\omega}{4}), (\ell_2) = (\frac{\omega}{4}, \frac{1}{2} - \frac{\omega}{4})$, appropriate for accounting the double-logarithmic contributions in the anti-parallel helicity fermion exchange, we encounter difficulties with the formulation of integral operators and wave functions on the plane. The corresponding kernels (2.4) (2.6) and wave functions (2.15) with these weights are not single valued irrespective to the particular choices of (d) and (ℓ_0).

We propose to describe the operators, wave functions and eigenvalues with these weights determined by the complex angular momentum ω as the analytic continuation from a series of corresponding objects with weights determined instead by even non-positive integers $-2m, -2m\ \ (m = 0, 1...)\ \to\ \omega$,

$$(\ell_{1,m}) = (\frac{1+m}{2}, -\frac{m}{2}),\quad (\ell_{2,m}) = (-\frac{m}{2}, \frac{1+m}{2}),$$
$$(\ell_{1,m} - \ell_{2,m}) = (\Delta_m) = (\frac{1}{2} + m, -\frac{1}{2} - m),\quad [\ell_{1,m} - \ell_{2,m}] = 1 + 2m,$$

$$(\ell_{1,m} + \ell_{2,m}) = (\frac{1}{2}, \frac{1}{2}). \quad (5.6)$$

The eigenvalues (2.17) decompose at $(d) = \pm(\Delta_m - \varepsilon)$ as

$$\begin{aligned}\lambda^{(\pm\Delta-\varepsilon)}_{(\ell_{1,m}),(\ell_{2,m}),(\ell_0)} &= \frac{1}{\varepsilon}\frac{1}{1+2m}\frac{\Gamma(\tilde{\ell}_0 \mp (\frac{1}{2}+m))\Gamma(1-\tilde{\ell}_0 \mp (\frac{1}{2}+m))}{\Gamma(1-\ell_0 \mp (\frac{1}{2}+m))\Gamma(\ell_0 \mp (\frac{1}{2})+m)} \\ &\{1 + \varepsilon\ [\ \chi_{\mp(\frac{1}{2}+m)}(\ell_0(1-\ell_0)) + \chi_{\mp(\frac{1}{2}+m}(\tilde{\ell}_0(1-\tilde{\ell}_0)) + \\ &\psi(1+2m) + \psi(2+2m) - 2\psi(1)] + \mathcal{O}(\varepsilon^2)\} \end{aligned} \quad (5.7)$$

We see that there is no obstacle to an analytic continuation of the expression on the right-hand side.

We calculate the exponents (2.6) with these weights and for $(d) = \pm(\Delta_m - \varepsilon)$ and define kernels in analogy to (5.2),

$$\begin{aligned}
K^{\frac{1}{2}+m,\delta} &= x_{12}\left(\frac{x_{12}}{x_{12}^*}\right)^m \frac{x_{1'2'}}{|x_{1'2'}^2|}\left(\frac{x_{1'2'}}{x_{1'2'}^*}\right)^m \delta^{(2)}(x_{21'})\,\frac{1}{|x_{12'}^2|}\left(\frac{x_{12'}}{x_{12'}^*}\right)^{-2m-1}, \\
K^{\frac{1}{2}+m,+} &= x_{12}\left(\frac{x_{12}}{x_{12}1^*}\right)^m \frac{x_{1'2'}}{|x_{1'2'}^2|}\left(\frac{x_{1'2'}}{x_{1'2'}^*}\right)^m \frac{1}{|x_{21'}^2|_+}\,\frac{1}{|x_{12'}^2|}\left(\frac{x_{12'}}{x_{12'}^*}\right)^{-2m-1}, \\
K^{-\frac{1}{2}+m,\delta} &= x_{12}^*\left(\frac{x_{12}}{x_{12}^*}\right)^{-m} \frac{x_{1'2'}^*}{|x_{1'2'}^2|}\left(\frac{x_{1'2'}}{x_{1'2'}^*}\right)^{-m} \delta^{(2)}(x_{21'})\,\frac{1}{|x_{12'}^2|}\left(\frac{x_{12'}}{x_{12'}^*}\right)^{2m+1}, \\
K^{\frac{1}{2}+m,+} &= x_{12}^*\left(\frac{x_{12}}{x_{12}^*}\right)^{-m} \frac{x_{1'2'}^*}{|x_{1'2'}^2|}\left(\frac{x_{1'2'}}{x_{1'2'}^*}\right)^{-m} \frac{1}{|x_{21'}^2|_+}\,\frac{1}{|x_{12'}^2|}\left(\frac{x_{12'}}{x_{12'}^*}\right)^{2m+1}
\end{aligned} \quad (5.8)$$

We have the decomposition of the operators $\hat{K}^{(\pm\Delta-\varepsilon)}_{(\ell_{1,m}),(\ell_{2,m})}$ in to the ones with the above kernels in analogy to (5.3). The series of operators

$$\hat{K}^{-(\frac{1}{2}+m),\delta}\ \hat{K}^{+(\frac{1}{2}+m)+}\ +\ \hat{K}^{+(\frac{1}{2}+m),\delta}\ \hat{K}^{-(\frac{1}{2}+m),+} \quad (5.9)$$

has the eigenvalues on the two-reggeon wave functions (2.15) correspondingly with the weights $\ell_{1,m}) = (\frac{1+m}{2}, -\frac{m}{2}), \quad \ell_{2,m}) = (\frac{m}{2}, \frac{1+m}{2}), (\ell_0)$

$$\begin{aligned}&\chi_{-\frac{1}{2}-m}(\tilde{\ell}_0(1-\tilde{\ell}_0))\ +\ \chi_{\frac{1}{2}+m}(\tilde{\ell}_0(1-\tilde{\ell}_0))\ +\ \chi_{\frac{1}{2}+m}(\ell_0(1-\ell_0)) + \\ &\chi_{-\frac{1}{2}-m}(\ell_0(1-\ell_0))\ + 2\psi(1+2m) + 2\psi(2+2m) - 4\psi(1). \end{aligned}\quad (5.10)$$

The case $m = 0$ is the one considered in the first part of this section. The series of wave functions with $(\ell_{1,m}), (\ell_{2,m})$ allows to describe the "dressed" two-fermion eigen-states with anti-parallel helicity and the series of operators (5.9) their interaction in the sense of the above analytic continuation to the complex ω from integers $-2m$. In [10] we have shown that the eigenvalues obtained from (5.10) after this continuation describe anti-parallel helicity fermion exchange in $\ln s$ accuracy.

6 Discussion

The perturbative Regge exchanges involving not only the leading gluonic reggeons are relevant in special semi-hard processes and contribute e.g. to the small x

behaviour of flavour non-singlet, spin or chiral-odd structure functions. The comparison of the reggeon interaction involving fermions to the standard BFKL case shows interesting common symmetry pattern.

Considering the generic conformal operators formulated in terms of conformal 4-point functions, determined by the weights $(\ell_1)(\ell_2)$ and a further parameter doublet (d), we have identified the particular operators describing the one-loop perturbative QCD reggeon interaction. In all cases the QCD reggeon interaction operator appear in the decomposition of the generic conformal operator at the singular values of the parameter $(d) = \pm(\ell_1 - \ell_2)$. In particular the BFKL one-loop kernel is reproduced in the dipole form [17], applicable to two-reggeon states described by wave functions vanishing at coinciding points (Möbius representation [22]).

The shift in the conformal spin proportional to the complex angular momentum accounting for the double-logarithmic contributions in the anti-parallel helicity fermion exchange channel would not result in single-valued integral kernels. We propose to describe the corresponding states and interaction operators by analytic continuation from a series of such objects with half-integer spins $s_m = [\ell_m] = \pm(\frac{1}{2} + m)$.

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